Two Examples of the EDM Storage Ring Lattice: Deuteron Storage Ring and Storage Ring for Ions with Z=50

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Outline of the talk

- Requirements to sensitivity of the EDM search
- Spin motion equations in the EDM storage ring
- Different sourses of the spin motion pertubations
- Possible approach to the EDM search experiment
- Conclusion

Requirements to Sensitivity of the EDM Search

$$d = \frac{Ze}{2m}\eta$$

Let: $d \approx 10^{-24} e{\cdot}\mathrm{cm}$

$$\frac{Z}{m} \simeq \begin{cases} 3.86 \cdot 10^{-11}\mathrm{cm} & e^+,\ e^- \\ 1.9 \cdot 10^{-13}\mathrm{cm} & \mu^+,\ \mu^- \\ 0.8 \cdot 10^{-14}\mathrm{cm} & {}^{133}_{53}\mathrm{I},\ \text{iodine ion} \end{cases}$$

$$\eta \simeq \begin{cases} 0.5 \cdot 10^{-13} & e^+,\ e^- \quad \text{Really, } \eta_e \le 10^{-16},\ d_e \le 10^{-27} e \cdot \mathrm{cm} \\ 1 \cdot 10^{-11} & \mu^+,\ \mu^- \\ 2.5 \cdot 10^{-10} & {}^{133}_{53}\mathrm{I} \end{cases}$$

$$\vec{\Omega}_{edm} = -\frac{Ze}{m}\eta\left\{\vec{E} + \vec{v} \times \vec{B}\right\}$$

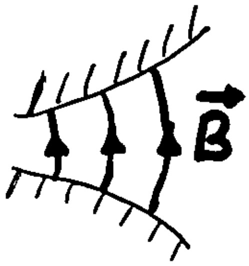
EDM spin tune is:

$$\nu_{edm} = \Omega_{edm}/\Omega_p$$

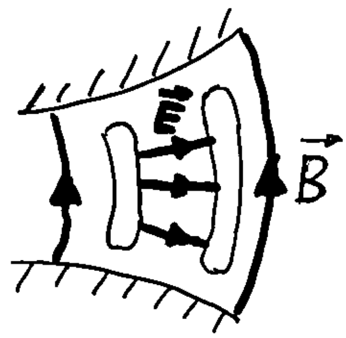
$$\nu_{edm} = \eta \cdot \sqrt{\gamma^2 - 1}$$

So, heavy ion system looks most promising for the search of the EDM due to its relatively large value of the η−factor!

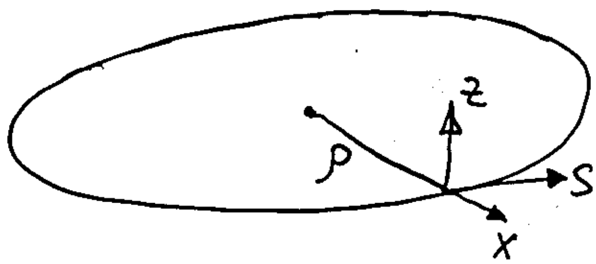
Weak Focusing Storage Ring
with Crossed Electric and Magnetic Fields



$n_B > 0$



$n_E > 1$



The field configuration:

$$\begin{cases} E_x &= E_0\,(1 - n_E\,\dfrac{x}{\rho}) \\ E_z &= -E_0\,(1 - n_E)\,\dfrac{z}{\rho} \end{cases}$$

$$\begin{cases} B_x &= -B_0\,n_B\,\dfrac{z}{\rho} \\ B_z &= B_0\,(1 - n_B\,\dfrac{x}{\rho}) \end{cases}$$

Main Lattice Parameters of the Ring

For the arbitrary ratio E_0/B_0 betatron tunes Q_x, Q_z are:

$$Q_x = \sqrt{\frac{\rho}{\rho_B}(1-n_B) + \frac{\rho}{\rho_E}(2-n_E) + \frac{\rho^2}{\rho_E^2\gamma_0^2}}$$

$$Q_z = \sqrt{\frac{\rho}{\rho_B}n_B + \frac{\rho}{\rho_E}(n_E-1)}$$

$$Q_x^2 + Q_z^2 = 1 + \frac{\rho^2}{\rho_E^2\gamma_0^2}$$

In specific case, when magnetic moment spin tune is equal to zero. they are:

$$Q_x = \sqrt{1 - n_B + \frac{\gamma_0^2}{1+a^{-1}}\left[1 + n_B - n_E + \frac{1}{1+a^{-1}}\right]}$$

$$Q_z = \sqrt{n_B - \frac{\gamma_0^2}{1+a^{-1}}(1 + n_B - n_E)}$$

$$Q_x^2 + Q_z^2 = 1 + \frac{\gamma_0^2}{(1+a^{-1})^2}$$

Momentum dispersion function is:

$$D_x \equiv \Delta x/(\Delta p/p) = \rho\frac{1+(1+a^{-1})^{-1}}{1 - n_B + (n_B - n_E)\gamma_0^2(1+a^{-1})^{-1}}$$

Here p is the dimensionless momentum:

$$p = \sqrt{\gamma^2 - 1}$$

For the discussed above numerical example ($\gamma = 1.1$, $a = 0.016$):

$$Q_x^2 + Q_z^2 = 1.0003$$

$$D_x = \rho\frac{1.01575}{1 - n_B + (n_B - n_E)1.019}$$

Spin motion in the EDM storage ring

$$\vec{\omega}_m = -\frac{Ze}{m}\cdot\left\{\left(a+\frac{1}{\gamma}\right)\vec{B} - a\frac{\gamma}{\gamma+1}\vec{v}(\vec{v}\vec{B}) - \left(a+\frac{1}{\gamma+1}\right)\vec{v}\times\vec{E}\right\}$$

$$\vec{\omega}_p = -\frac{Ze}{m}\left\{\frac{\vec{B}}{\gamma} - \frac{\gamma}{\gamma^2-1}\vec{v}\times\vec{E}\right\}$$

$$\vec{B} = \vec{B}_\perp + \vec{B}_\parallel \qquad \vec{B}_\perp\vec{v} = 0$$

$$\vec{\omega}_{p\perp} = -\frac{Ze}{m}\left\{\frac{\vec{B}_\perp}{\gamma} - \frac{\gamma}{\gamma^2-1}\vec{v}\times\vec{E}\right\}$$

$$\vec{\omega}_m - \vec{\omega}_{p\perp} + \vec{\omega}_{edm} = -\frac{Ze}{m}\left\{a\vec{B}_\perp + \frac{1+a}{\gamma}\vec{B}_\parallel - \left(a-\frac{1}{\gamma^2-1}\right)\vec{v}\times\vec{E}\right\} -$$
$$-\frac{Ze}{m}\cdot\eta\cdot\left\{\vec{E}+\vec{v}\times\vec{B}\right\}$$

$$\omega_m - \omega_{p\perp} = 0 \rightarrow \quad aB + \left(a-\frac{1}{\gamma^2-1}\right)vE = 0$$

Some useful expressions:

$$\omega_p = -\frac{Ze}{m}\cdot\frac{B}{\gamma}\cdot\frac{1+a}{1-(\gamma^2-1)a}$$

$$\omega_{edm} = -\frac{Ze}{m}\cdot\eta\cdot E\cdot\frac{1+\frac{1}{a}}{\gamma^2} = -\frac{Ze}{m}\cdot\eta\cdot v\cdot B\frac{1+a}{1-(\gamma^2-1)a}$$

$$V_{edm} = \frac{\omega_{edm}}{\omega_p} = \eta\cdot v\cdot\gamma$$

Spin Tune Spread

Spin tune momentum dependence (spin tune chromaticity):

$$C = \frac{\Delta\nu_z}{\Delta p/p} = a\gamma_0[1 - a(\gamma_0^2 - 1)] \times$$
$$\times \left\{ \frac{1 + (2+a)(\gamma_0^2 - 1)}{\gamma_0^2[1 - a(\gamma_0^2 - 1)]} - \frac{(n_B - n_E)[1 + (1 + a^{-1})^{-1}]}{1 - n_B + (n_B - n_E)\gamma_0^2(1 + a^{-1})^{-1}} \right\}$$

Effects on the spin tune from the horizontal betatron oscillations:

$$\Delta\nu_z = -\frac{x}{\rho} a\gamma_0 \frac{1 - a(\gamma_0^2 - 1)}{1 + a} \left\{ n_B - n_E + \frac{\gamma_0^2}{1 + a^{-1}} \left[\frac{2}{1 - a(\gamma_0^2 - 1)} - \frac{1}{\gamma_0^2} \right] \right\}$$

and from the vertical betatron oscillations:

$$\Delta\nu_x = \frac{z}{\rho} a\gamma_0 \frac{1 - a(\gamma_0^2 - 1)}{1 + a} (1 - n_E - n_B)$$

Here x and z are the deviations from the equilibrium closed orbit.

In principle, choosing appropriate values of the field indexes n_B and n_E we can have both expressions in the braces vanishing, but in reality only the spin tune chromaticity gives a large effect and the value of n_B can be chosen more or less free, appropriate for betatron stability, and the cancellation condition $(C = 0)$ may be expressed in the form:

$$n_E = n_B + \frac{1 + (2 + a)(\gamma_0^2 - 1)}{1 - 3a(\gamma_0^2 - 1)} \frac{n_B - 1}{\gamma_0^2}$$

Note that $n_E = n_B = 1$ is one of the solutions of the above equation, but $n_B = 1$ is not acceptable for betatron stability. Substituting $\gamma_0 = 1.1$, and $a = .016$ one get:

$$n_E = 2.1883 n_B - 1.1883$$

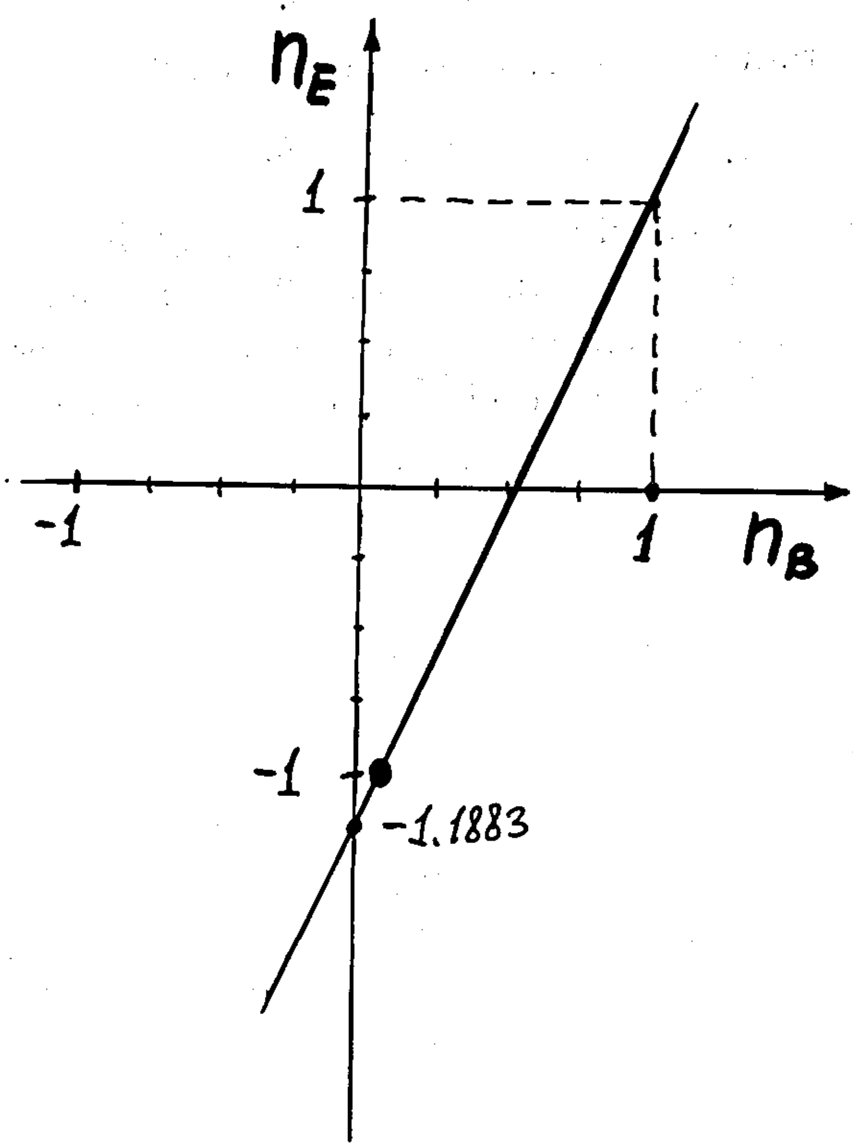
The corresponding value of the closed orbit shift is:

$$z = \rho \frac{a\gamma^2}{n_B(1 + a(\gamma^2 - 1))} \Delta\theta \simeq 10^{-10} \rho / n_B$$

Substituting $\rho = 3.74$ m, $n_B = 0.1$ one get:

$$z \simeq 3.74 \text{ nm}$$

To measure orbital movement we can apply the increased electric field: say not 24 kV/cm but ± 100 kV/cm! Thus, sensitivity of the order of 10 nanometers of the beam position monitors will be sufficient for monitoring of the electrodes alignment.



n_E
1
-1
-1
1
n_B
-1.1883

Effects on the Spin Motion from the Closed Orbit Distortions

Purely vertical or purely horizontal closed orbit distortions are not dangerous, because of vanishing of the integral of the distortion over the turn (except for the vertical zero harmonic, which generates the magnetic moment precession around x-axis). But in combination vertical and horizontal harmonics with equal amplitude A and integer harmonic number k being shifted in phase by ψ produce non-zero spin rotation around longitudinal axis:

$$\Delta\nu_x(\theta) = A\cos k\theta$$

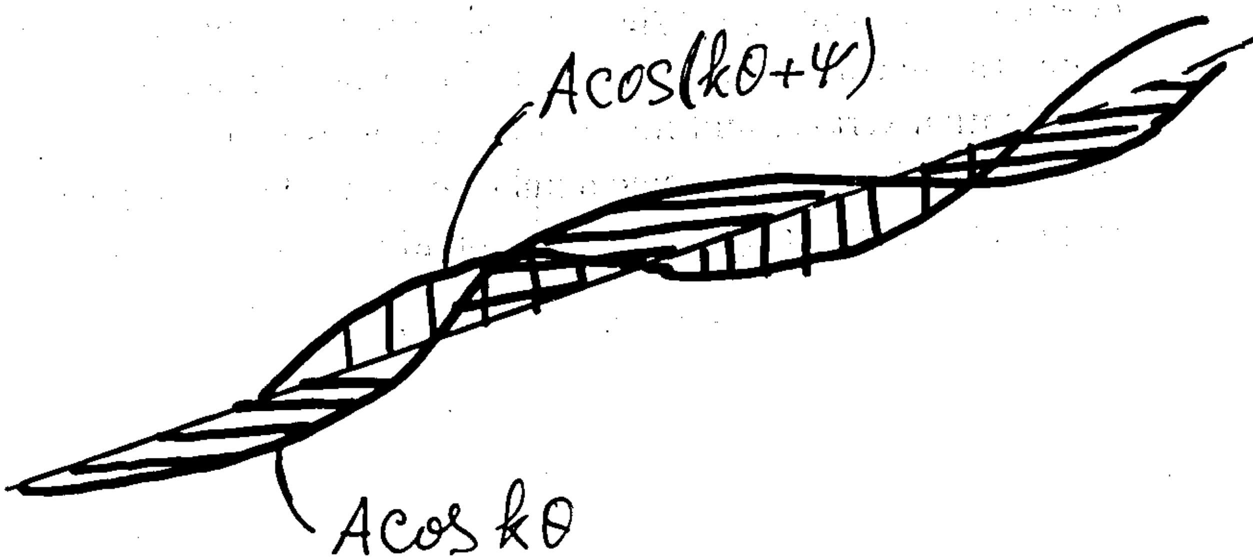
$$\Delta\nu_z(\theta) = A\cos(k\theta + \psi)$$

$$\Delta\nu_s \simeq \frac{A^2}{2k}\sin\psi$$

This effect is simply a consequence of the noncommutativity of rotations around two orthogonal axes. So, if: $\nu_{edm} \simeq 10^{-10}$, then $A^2/2k \leq 10^{-10}$, or (for $k = 1$, $\psi = \pi/2$):

$$A \leq 1.4 \cdot 10^{-5}$$

Below it will be shown that choosing appropriately values of electric and magnetic field indexes n_E and n_B it is possible to make horizontal spin tune insensitive to the radial orbit deviation thus eliminating the circular component of the spin perturbation.



$A\cos(k\theta+\psi)$
$A\cos k\theta$

Discussion of Accuracy Requirements to the Field Configuration

Let us estimate how severe are the restrictions on the spin chromaticity value. Spin tune spread should not exceed $\nu_{edm} \simeq 10^{-10}$, otherwise decoherence of the spin directions will happen earlier than the spin makes one turn due to the EDM precession. Two numerical examples:

$$\Delta p/p \simeq 10^{-3} \rightarrow C \leq 10^{-7} \rightarrow \delta n_E \simeq \pm 10^{-5}$$

$$\Delta p/p \simeq 10^{-6} \rightarrow C \leq 10^{-4} \rightarrow \delta n_E \simeq \pm 10^{-2}$$

Small value of the momentum spread in the ion beam can be achieved using electron precooling. As a result, less accurate electric and magnetic field geometry will be needed.

As it was shown above, restrictions on the betatron tune spread $\Delta\nu_z$ and $\Delta\nu_x$ are of the order:

$$\Delta\nu_{x,z} \leq 1.4 \cdot 10^{-5}$$

Since $\Delta\nu_{x,z} \simeq a\gamma_0(x, z/\rho)$ (roughly), and having $a\gamma_0 \simeq 1.8 \cdot 10^{-2}$ in our example, one gets:

$$\frac{x, z}{\rho} \leq 10^{-3}$$

This restrictions on the amplitudes of betatron oscillations and closed orbit distortions are more or less relax.

How Precise the Compensation of the Magnetic Moment Precession Should Be Done?

In order not to have precession around vertical axis, relative stability of the ratio E_0/B_0 should be of the order

$$\nu_{edm}/a\gamma[1 - a(\gamma^2 - 1)] \simeq 10^{-8}.$$

Nonzero radial component of the spin tune ν_x may simulate or disguise the EDM effect! Let's consider effect from the constant vertical component of the electric field E_z (such field may be generated by inclination of the potential electrodes). Radial components of the magnetic moment precession frequency and frequency of the momentum precession are:

$$\Omega_x = -\frac{Ze}{m}[aB_x + (a - \frac{1}{\gamma^2 - 1})vE_z]$$

$$(\Omega_p)_x = -\frac{Ze}{\gamma m}\left[B_x + \frac{\gamma^2}{\gamma^2 - 1}vE_z\right]$$

Taking into account that $(\Omega_p)_x = 0$, $B_x = -B_0 n_B z/\rho$. $\Omega_{edm} = -Zem^{-1}\eta E_0\gamma^{-2}(1 + a^{-1})$ one finds:

$$\Omega_x = \frac{Ze}{m}vE_z\frac{1 + a}{\gamma^2 - 1}$$

$$\frac{\Omega_x}{\Omega_{edm}} = -\frac{a}{\eta v}\frac{E_z}{E_0}$$

$$\frac{z}{\rho} = \frac{a\gamma^2}{n_B(1 - a(\gamma^2 - 1))}\frac{E_z}{E_0}$$

But $E_z = \Delta\theta \cdot E_0$, hence:

$$\underline{\Delta\theta \leq \eta\frac{v}{a} \simeq 10^{-8}}$$

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	Deuteron $^{2}_{1}H$			$^{133}_{53}I$			Helium like ion $^{139}_{55}Cs$		
Z	1	-"-	-"-	53	-"-	-"-	53	-"-	-"-
A	2	-"-	-"-	133	-"-	-"-	139	-"-	-"-
m (GeV)	1.87	-"-	-"-	130	-"-	-"-	130	-"-	-"-
a	−0.143	-"-	-"-	0.016	-"-	-"-	0.002	-"-	-"-
v	0.2	0.4	0.5	0.68	0.75	0.8	0.88	0.9	0.91
γ	1.0206	1.091	1.155	1.364	1.511	1.667	2.105	2.294	2.412
BR (T·m)	1.273	2.722	3.6	7.588	9.277	10.9	15.16	16.89	17.958
T/A (GeV)	0.0193	0.085	0.145	.355	0.5	0.65	1.034	1.21	1.32
E (kV/cm)	−40	−60	−100	40	60	100	40	40	60
B (T)	.45	0.3	0.366	.65	.714	0.91	1.7	1.4	1.87
ρ_E (m)	−19.1	−54.4	−54	387	347.9	262	1000	1140	817
ρ_B (m)	2.828	9.02	9.83	11.68	12.99	11.98	8.93	12.1	9.6
ρ (m)	3.32	10.81	12.02	11.34	12.52	11.45	8.85	11.98	9.5
η	$2\cdot10^{-10}$	-"-	-"-	$2.5\cdot10^{-10}$	$2.5\cdot10^{-10}$	$2.5\cdot10^{-10}$	$2.5\cdot10^{-10}$	-"-	-"-
$\Delta\theta$ (nrad)	2.8	0.56	0.7	10.6	11.7	12.5	110	112.5	114
Z (nm) ($\beta=0.1$)	1.4	10	15.3	36.3	54.8	65.5	87	143	127
η_{edm}	$0.4\cdot10^{-10}$	$0.87\cdot10^{-10}$	$1.15\cdot10^{-10}$	$.23\cdot10^{-9}$	$.28\cdot10^{-9}$	$.333\cdot10^{-9}$	$0.46\cdot10^{-9}$	$0.516\cdot10^{-9}$	$.55\cdot10^{-9}$

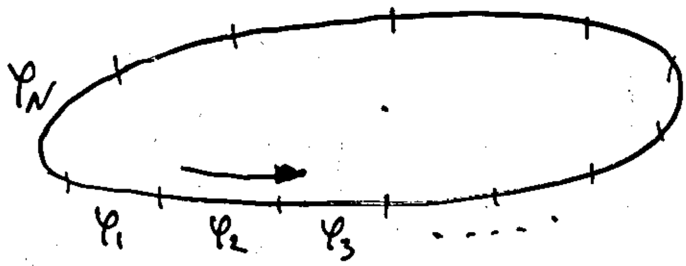
General properties of a spin motion in a storage ring

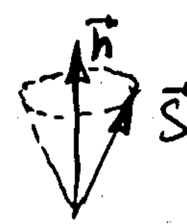
[Ya. Derbenev, A. Kondratenko, A. Skrinsky, 1970]

One turn map:
Spin precess (turn by turn) around some direction $\vec{n}(\theta)$ with a frequency ν, which is <u>independent</u> on azimuth θ.

$$\frac{\vec{\omega}(\theta)}{\omega_p} = \nu \vec{n}(\theta)$$

$\vec{S} \cdot \vec{n} = \text{const}$!





$$M = \prod_k \left[\cos\frac{\varphi_k}{2} - i\cdot(\vec{\sigma}\vec{n}_k)\sin\frac{\varphi_k}{2}\right]$$

$$M = \cos(\pi\nu) - i(\vec{\sigma}\vec{n})\sin(\pi\nu)$$

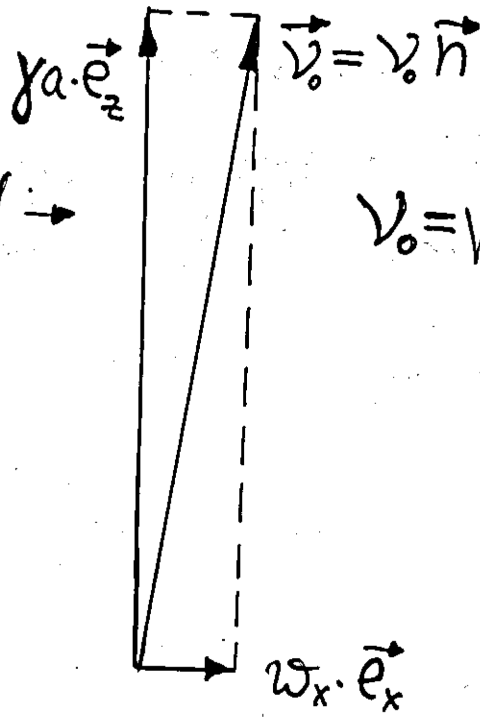
$$\begin{cases} \cos(\pi\nu) = \frac{1}{2}\,\mathrm{Tr}(M) \\ \vec{n} = \frac{i}{\sin\pi\nu}\cdot\frac{1}{2}\,\mathrm{Tr}(\vec{\sigma}M) \end{cases}$$

Spin precession without electric field and with it

No electric field →

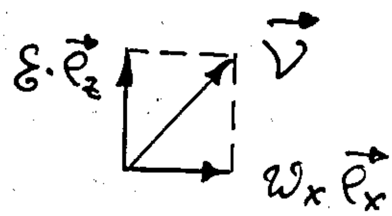
$\gamma a \simeq 10^{-2}$

$\omega_x \simeq 10^{-10}$



$$\nu_0 = \sqrt{\gamma^2 a^2 + \omega_x^2}$$

With the electric field →



$$\nu = \sqrt{\varepsilon^2 + \omega_x^2}$$

$$\varepsilon = \gamma a \cdot \frac{E_0 - E}{E_0}$$

$\varepsilon \simeq 10^{-9}$ if $\frac{E_0 - E}{E} \simeq 10^{-7}$

Due to low value of ε ν become sensitive to transversal components ω_x.

Conclusion

- Instead of measuring $\dot{S}_z$ we suggest to do a field mapping by studing of a distribution of $\vec{n}$ around the ring.
- Subsequently we shall correct the field errors, aiming to minimize field distortions to a level of the EDM scale.
- Proper alighnment of the electrodes relative to the $\vec{B}$ field should be proved by measuring of the vertical orbit displacement (when switching on the electric field) or by minimizing of the amplitude of a vertical betatron oscillations when applying the resonance electric RF-field.
- Synchrotron motoon will substancialy increase the spin decoherence time, but restrictions to the alighnment of a cavity should be studed additionaly.